Published by Institute of Physics Publishing for SISSA/ISAS

Received: *August 2, 2004*
Revised: *October 4, 2004*
Accepted: *November 28, 2004*

New ways to soft leptogenesis

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Abstract: Soft supersymmetry breaking terms involving heavy singlet sneutrinos provide new sources of lepton number violation and of CP violation. In addition to the CP violation in mixing, investigated previously, we find that 'soft leptogenesis' can be generated by CP violation in decay and in the interference of mixing and decay. These additional ways to leptogenesis can be significant for a singlet neutrino Majorana mass that is not much larger than the supersymmetry breaking scale, $M \lesssim 10^2 m_{\rm SUSY}$. In contrast to CP violation in mixing, for some of these new contributions the sneutrino oscillation rate can be much faster than the decay rate, so that the bilinear scalar term need not be smaller than its natural scale.

Keywords: Baryogenesis, Supersymmetric Standard Model, Neutrino Physics, CP violation.

JHEP11(2004)080

Contents

1. Introduction

The evidence for neutrino masses at a scale $\sim 10^{-2}$ eV makes a convincing case for the seesaw mechanism [1]–[4]: The existence of singlet neutrinos with Majorana masses and with Yukawa couplings to active neutrinos becomes very plausible. The physics of these heavy neutrinos can provide all the necessary ingredients for baryogenesis [5]: $B - L$ is violated by the Majorana masses, CP is likely to be violated in the neutrino Yukawa couplings and, for small enough Yukawa couplings, the heavy neutrinos decay out of equilibrium. Thus, leptogenesis [6], the dynamical generation of lepton asymmetry through the decays of heavy singlet Majorana neutrinos, becomes an attractive solution to the puzzle of the baryon asymmetry.

The seesaw mechanism introduces a new scale, M, the mass scale of the singlet neutrinos. Since this scale must be much higher than the electroweak breaking scale, $M \gg \Lambda_{\rm EW}$, a huge amount of fine-tuning is required within the framework of the Standard Model extended to include singlet neutrinos (SM+N) to keep the low Higgs mass. This situation provides further motivation to consider the supersymmetric extension of the model (SSM+N). Then, leptogenesis is induced in both singlet neutrino and singlet sneutrino decays. The results are modified by factors of order one, but the basic mechanism and the order of magnitude of the asymmetry remain very much the same as in the non-supersymmetric version.

Supersymmetry must, however, be broken. In addition to the soft supersymmetry breaking terms of the SSM, there are now terms that involve the singlet sneutrinos $\widetilde{N}$, in particular, bilinear (B) and trilinear (A) scalar couplings. These terms provide yet another source of lepton number violation and of CP violation. One may ask whether these terms can play a significant role in leptogenesis. One finds that for a certain range of parameters, the soft breaking terms play a significant role, and may even be dominant in leptogenesis [7, 8]. This scenario has been termed 'soft leptogenesis.' (For related work, see [9]–[13].)

In [7] we investigated soft leptogenesis related to CP violation in mixing (a leptonic analog of $\mathrm{Re}(\epsilon)_{K\to\pi\ell\nu}$). In this work, we present all the contributions to the lepton asymmetry that arise in this scenario. The contribution considered in [7] dominates over the other contributions in a large part of the parameter space. Yet, if the scale M is relatively low, other contributions, related to CP violation in the interference of decays with and without mixing (a leptonic analog of $S_{B\to\psi K}$), and to CP violation in decay (a leptonic analog of $\mathrm{Re}(\epsilon')_{K\to\pi\pi}$), play a significant role.

The plan of this paper is as follows. In section 2 we derive exact expressions for the singlet sneutrino decay rates into final (s)leptons in terms of mixing and decay amplitudes. In section 3 we present our model, that is the supersymmetric standard model extended to include singlet neutrinos (SSM+N) and express the mixing and decay amplitudes in terms of the model parameters. Our main results are obtained in sections 4 and 5. In section 4 we evaluate the lepton asymmetry in terms of the model parameters and, in particular, assuming hierarchy between the supersymmetry breaking scale and the mass scale of the singlet sneutrinos, find the potentially leading contributions. In section 5 we estimate the size of the various contributions and find the regions in the SSM+N parameter space where these contributions can account for the observed baryon asymmetry. We summarize our results and draw further conclusions in section 6. Additional points are made in two appendices. In appendix A we explicitly prove that the consideration of three body final states does not change the picture. In appendix B we discuss the possibility of naturally obtaining a small B term for the singlet sneutrinos.

2. Mixing and decay

We would like to calculate the CP-violating lepton asymmetry:

$$\varepsilon_\ell \equiv \frac{\Gamma(\widetilde{L})+\Gamma(L)-\Gamma(\widetilde{L}^\dagger)-\Gamma(\overline{L})}{\Gamma(\widetilde{L})+\Gamma(L)+\Gamma(\widetilde{L}^\dagger)+\Gamma(\overline{L})}\,, \tag{2.1}$$

where $\Gamma(X)$ is the time-integrated decay rate into a final state with a leptonic content X. Here $L(\overline{L})$ is the (anti)lepton doublet and $\widetilde{L}(\widetilde{L}^\dagger)$ is the (anti)slepton doublet.

A crucial role in our results is played by the $\widetilde{N}-\widetilde{N}^\dagger$ mixing amplitude,

$$\langle \widetilde{N}|\mathcal{H}|\widetilde{N}^\dagger\rangle = M_{12} - \frac{i}{2}\Gamma_{12}\,, \tag{2.2}$$

which induces mass and width differences,

$$x \equiv \frac{\Delta M}{\Gamma} \equiv \frac{M_H - M_L}{\Gamma}\,, \qquad y \equiv \frac{\Delta\Gamma}{2\Gamma} \equiv \frac{\Gamma_H - \Gamma_L}{2\Gamma}\,, \tag{2.3}$$

(Γ is the average width) between the two mass eigenstates, the heavy $|\widetilde{N}_H\rangle$ and the light $|\widetilde{N}_L\rangle$,

$$|\widetilde{N}_{L,H}\rangle = p|\widetilde{N}\rangle \pm q|\widetilde{N}^\dagger\rangle\,. \tag{2.4}$$

The ratio q/p depends on the mixing amplitude ratio:

$$\left(\frac{q}{p}\right)^2 = \frac{2M_{12}^* - i\Gamma_{12}^*}{2M_{12} - i\Gamma_{12}}\,. \tag{2.5}$$

For each final state X, we define a pair of amplitudes and a quantity λ_X involving the amplitude ratio and the mixing amplitudes:

$$A_X = \langle X|\mathcal{H}|\widetilde{N}\rangle\,, \qquad \overline{A}_X = \langle X|\mathcal{H}|\widetilde{N}^\dagger\rangle\,, \qquad \lambda_X = \frac{q}{p}\frac{\overline{A}_X}{A_X}\,. \tag{2.6}$$

Defining $|\widetilde{N}(t)\rangle$ and $|\widetilde{N}^\dagger(t)\rangle$ to be the states that evolve from purely $|\widetilde{N}\rangle$ and $|\widetilde{N}^\dagger\rangle$, respectively, at time $t=0$, we obtain the following time-dependent decay rates into a final state X:

$$\begin{aligned}
\Gamma(\widetilde{N}(t)\to X) &= \mathcal{N}_X|A_X|^2 e^{-\Gamma t}\Bigg[\frac{1+|\lambda_X|^2}{2}\cosh\frac{\Delta\Gamma t}{2} + \frac{1-|\lambda_X|^2}{2}\cos(\Delta M\ t) + \\
&\qquad + \mathrm{Re}\,\lambda_X \sinh\frac{\Delta\Gamma t}{2} - \mathrm{Im}\,\lambda_X \sin(\Delta M\ t)\Bigg]\,,\\
\Gamma(\widetilde{N}^\dagger(t)\to X) &= \mathcal{N}_X|A_X|^2 \left|\frac{p}{q}\right|^2 e^{-\Gamma t}\Bigg[\frac{1+|\lambda_X|^2}{2}\cosh\frac{\Delta\Gamma t}{2} - \frac{1-|\lambda_X|^2}{2}\cos(\Delta M\ t) + \\
&\qquad + \mathrm{Re}\,\lambda_X \sinh\frac{\Delta\Gamma t}{2} + \mathrm{Im}\,\lambda_X \sin(\Delta M\ t)\Bigg]\,,
\end{aligned} \tag{2.7}$$

where $\mathcal{N}_X$ is a phase space factor. Summing over the initial states, $\widetilde{N}$ and $\widetilde{N}^\dagger$, we obtain the following four time-integrated decay rates (in arbitrary units):

$$\begin{aligned}
\Gamma(\widetilde{L}) &= \mathcal{N}_s|A_{\widetilde{L}}|^2\Bigg[\frac{(1+|p/q|^2)(1+|\lambda_{\widetilde{L}}|^2)}{2(1-y^2)} + \frac{(1-|p/q|^2)(1-|\lambda_{\widetilde{L}}|^2)}{2(1+x^2)} + \\
&\qquad + \frac{y(1+|p/q|^2)\,\mathrm{Re}\,\lambda_{\widetilde{L}}}{1-y^2} - \frac{x(1-|p/q|^2)\,\mathrm{Im}\,\lambda_{\widetilde{L}}}{1+x^2}\Bigg]\,,\\
\Gamma(\widetilde{L}^\dagger) &= \mathcal{N}_s|\bar{A}_{\widetilde{L}^\dagger}|^2\Bigg[\frac{(1+|q/p|^2)(1+|\lambda_{\widetilde{L}^\dagger}|^{-2})}{2(1-y^2)} + \frac{(1-|q/p|^2)(1-|\lambda_{\widetilde{L}^\dagger}|^{-2})}{2(1+x^2)} + \\
&\qquad + \frac{y(1+|q/p|^2)\,\mathrm{Re}\,\frac{1}{\lambda_{\widetilde{L}^\dagger}}}{1-y^2} - \frac{x(1-|q/p|^2)\,\mathrm{Im}\,\frac{1}{\lambda_{\widetilde{L}^\dagger}}}{1+x^2}\Bigg]\,,\\
\Gamma(\overline{L}) &= \mathcal{N}_f|A_{\overline{L}}|^2\Bigg[\frac{(1+|p/q|^2)(1+|\lambda_{\overline{L}}|^2)}{2(1-y^2)} + \frac{(1-|p/q|^2)(1-|\lambda_{\overline{L}}|^2)}{2(1+x^2)} + \\
&\qquad + \frac{y(1+|p/q|^2)\,\mathrm{Re}\,\lambda_{\overline{L}}}{1-y^2} - \frac{x(1-|p/q|^2)\,\mathrm{Im}\,\lambda_{\overline{L}}}{1+x^2}\Bigg]\,,\\
\Gamma(L) &= \mathcal{N}_f|\overline{A}_L|^2\Bigg[\frac{(1+|q/p|^2)(1+|\lambda_L|^{-2})}{2(1-y^2)} + \frac{(1-|q/p|^2)(1-|\lambda_L|^{-2})}{2(1+x^2)} + \\
&\qquad + \frac{y(1+|q/p|^2)\,\mathrm{Re}\,\frac{1}{\lambda_L}}{1-y^2} - \frac{x(1-|q/p|^2)\,\mathrm{Im}\,\frac{1}{\lambda_L}}{1+x^2}\Bigg]\,.
\end{aligned} \tag{2.8}$$

Using these four decay rates, we can obtain an exact expression for ε_ℓ defined in eq. (2.1).

3. The SSM+N

Since we are interested in the effects of the soft supersymmetry breaking couplings, we work in a simplified single generation model. The relevant superpotential terms are

$$W = Y\epsilon_{\alpha\beta}L_\alpha N H_\beta + \frac{1}{2}MNN\,, \tag{3.1}$$

where L is the supermultiplet containing the left-handed lepton doublet fields, N is the superfield whose left-handed fermion is the SU(2) $\times$ U(1)-singlet $\overline{\nu}_L$, and H is the Higgs doublet (usually denoted by H_2). The relevant soft supersymmetry breaking terms in the lagrangian are the following:

$$\mathcal{L}_{\rm SSB} = -\left(m_2\lambda_2^a\lambda_2^a + A\epsilon_{\alpha\beta}\widetilde{L}_\alpha\widetilde{N}H_\beta + B\widetilde{N}\widetilde{N} + {\rm h.c.}\right). \tag{3.2}$$

Here λ_2^a $(a = 1,2,3)$ are the SU$(2)_L$ gauginos, $\widetilde{N}, \widetilde{L}, H$ are scalar fields (and N, L, h are their fermionic superpartners). The U$(1)_Y$ gaugino, λ_1, would give effects that are similar to those of λ_2 and can be included in a straightforward way.

The lagrangian derived from eqs. (3.1) and (3.2) has two independent physical CP violating phases:

$$\begin{aligned}\phi_N &= \arg(AMB^*Y^*)\,,\\ \phi_W &= \arg(m_2MB^*)\,.\end{aligned} \tag{3.3}$$

These phases give the CP violation that is necessary to dynamically generate a lepton asymmetry. If we set the lepton number of N and $\widetilde{N}$ to -1, so that Y and A are lepton number conserving, the two couplings M and B violate lepton number by two units. Thus processes that involve Y or A, and M or B, would give the lepton number violation that is necessary for leptogenesis.

There are several dimensionful parameters in (3.1) and (3.2). Of these M is supersymmetry conserving and all other are supersymmetry breaking. We assume the following hierarchies:

$$\epsilon_{\rm S} \equiv \frac{m_{\rm SUSY}}{M} \ll 1\,, \tag{3.4}$$

where $m_{\rm SUSY}$ is the supersymmetry breaking scale in the SSM+N (we take $m_{\rm SUSY} \sim 1$ TeV), and, unless otherwise stated,

$$|m_2| \sim \left|\frac{A}{Y}\right| \sim \left|\frac{B}{M}\right| \sim m_{\rm SUSY}\,. \tag{3.5}$$

We also assume that $|Y| \ll 1$, as is required by the condition of out-of-equilibrium decay (see eq. (5.6)).

We can evaluate the various parameters of eq. (2.8) in terms of the lagrangian parameters of eqs. (3.1) and (3.2). The singlet sneutrino decay width is given, for $|MY| \gg |A|$, by

$$\Gamma = \frac{|MY^2|}{4\pi}\,. \tag{3.6}$$

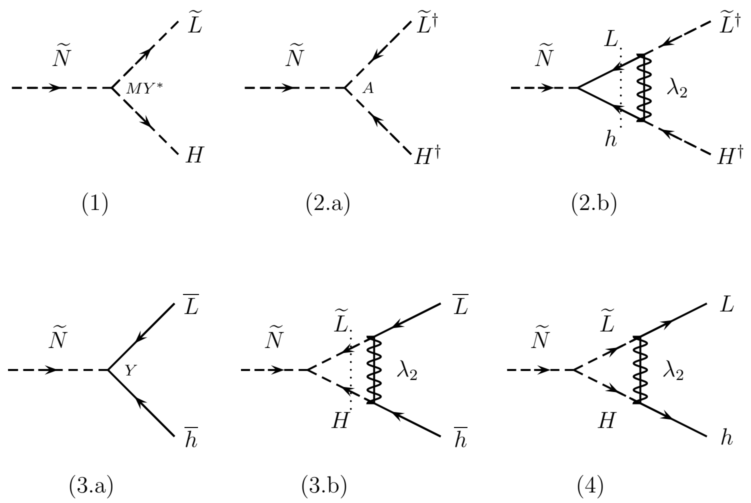

Figure 1: Two-body decay diagrams of a singlet sneutrino.

For the mixing parameters, we obtain

$$
\begin{aligned}
x &= \frac{2|B|}{|M|\Gamma} = \frac{8\pi|B|}{|MY|^2}\,, \\
y &= \left|\frac{A}{MY}\right|\cos\phi_N - \left|\frac{B}{M^2}\right|, \\
\left|\frac{q}{p}\right| &= \left(1 + \frac{2|AMY/(4\pi B)|\sin\phi_N}{1 - |AMY/(4\pi B)|\sin\phi_N + \frac{1}{4}|AMY/(4\pi B)|^2}\right)^{1/4}.
\end{aligned}
\tag{3.7}
$$

As concerns the decay amplitudes, CPT guarantees the following relation:

$$
|A_{\widetilde{L}}|^2 + |A_{\widetilde{L}^\dagger}|^2 + |A_{\overline{L}}|^2 + |A_L|^2 = |\overline{A}_{\widetilde{L}}|^2 + |\overline{A}_{\widetilde{L}^\dagger}|^2 + |\overline{A}_{\overline{L}}|^2 + |\overline{A}_L|^2\,. \tag{3.8}
$$

We consider only two body final states, since three body (or higher) states give only small corrections, as shown in appendix A. In Fig. 1 we show the relevant diagrams (including the dominant one loop corrections) for the four two body final states: (1) $\widetilde{L}H$, (2) $\widetilde{L}^\dagger H^\dagger$, (3) $\overline{Lh}$, and (4) Lh. The amplitudes are given by (we use an $=$ sign when the difference between the absolute values of two CP conjugate amplitudes is negligible and an $\approx$ sign when it is not)

$$
\begin{aligned}
|A_{\widetilde{L}}| &= |\overline{A}_{\widetilde{L}^\dagger}| = |MY|\,, \\
|A_{\overline{L}}| &\approx |\overline{A}_L| \approx |MY|\,, \\
|\overline{A}_{\widetilde{L}}| &\approx |A_{\widetilde{L}^\dagger}| \approx |A|\,, \\
|\overline{A}_{\overline{L}}| &= |A_L| = \frac{3\alpha_2}{4}|m_2Y|\sqrt{f_1^2+f_2^2} = \mathcal{O}\left(\alpha_2|m_2Y|\right), \\
|A_{\overline{L}}| - |\overline{A}_L| &= -\frac{3\alpha_2}{2}\left|\frac{m_2A}{M}\right| f_1\sin(\phi_W-\phi_N) = \mathcal{O}\left(\alpha_2\left|m_2\frac{A}{M}\right|\right), \\
|\overline{A}_{\widetilde{L}}| - |A_{\widetilde{L}^\dagger}| &= \frac{3\alpha_2}{2}|m_2Y|f_1\sin(\phi_W-\phi_N) = \mathcal{O}\left(\alpha_2|m_2Y|\right),
\end{aligned}
\tag{3.9}
$$

where $\alpha_2 = g_2^2/(4\pi)$ is the weak coupling constant and where we define

$$f_1 = \ln\left(\frac{M^2+m_2^2}{m_2^2}\right),$$
$$f_2 = \mathrm{Li}_2\left(\frac{2}{1-\sqrt{1+4m_2^2/M^2}}\right) + \mathrm{Li}_2\left(\frac{2}{1+\sqrt{1+4m_2^2/M^2}}\right). \tag{3.10}$$

These expressions assume, for simplicity, that $m_2 > m_{\tilde{L}}, m_H$, thus neglecting corrections proportional to $m_{\tilde{L}}$ and m_H. The function $\mathrm{Li}_2(z) \equiv \int_z^0 \frac{\ln(1-t)dt}{t}$ is the dilogarithm function. For the relevant strong and weak phases, we obtain

$$\phi_s \equiv \frac{1}{2}\arg\left(\lambda_{\tilde{L}}\lambda_{\tilde{L}^\dagger}\right) = -\phi_N\,,$$
$$\phi_f \equiv \frac{1}{2}\arg\left(\lambda_{\overline{L}}\lambda_{L}\right) = -\phi_W\,,$$
$$\sin\delta_s \equiv \sin\frac{\arg(\lambda_{\tilde{L}}\lambda_{\tilde{L}^\dagger}^{-1})}{2} = \frac{3\alpha_2}{4}\left|\frac{m_2 Y}{A}\right| f_1 = \mathcal{O}(\alpha_2)\,,$$
$$\sin\delta_f \equiv \sin\frac{\arg(\lambda_{\overline{L}}\lambda_{L}^{-1})}{2} = \frac{f_1}{\sqrt{f_1^2+f_2^2}} = \mathcal{O}(1)\,. \tag{3.11}$$

Note that there are several relations between decay amplitudes to final scalars and to final fermions. These relations have to be taken into account when evaluating the asymmetry. First, in the supersymmetric limit we have $|A_{\tilde{L}}| = |A_{\overline{L}}|$. Second, we have $|\overline{A}_{\tilde{L}}/A_{\tilde{L}}|\sin\delta_s = |\overline{A}_{\overline{L}}/A_{\overline{L}}|\sin\delta_f$. Similar relations hold for the CP conjugate amplitudes.

4. The leading contributions to ε_ℓ

Many terms that contribute to the lepton asymmetry are small and can be neglected. The small parameters that play a role are the ratio ϵ_{S}, the weak coupling constant α_2, and the Yukawa coupling Y. The dependence on the Yukawa coupling enters either via the combination $|A|/|MY|$ which, as can be seen from eq. (3.5), is taken to be of order ϵ_{S}, or via the x parameter evaluated in eq. (3.7). The x parameter can be small or large but, since $x \sim 8\pi\epsilon_{\mathrm{S}}/|Y|^2$, we take $x \gg \epsilon_{\mathrm{S}}$. (Some of the contributions that we consider are significant only for $B \ll \epsilon_{\mathrm{S}}M^2$ and, consequently, $x \ll 8\pi\epsilon_{\mathrm{S}}/|Y|^2$. In these cases, however, $x \sim 1$ is required, so that $x \gg \epsilon_{\mathrm{S}}$ is still valid.) We keep the x dependence explicit.

We identify several interesting contributions to ε_ℓ. We write down only the potentially leading contributions and neglect terms that are suppressed by higher powers of ϵ_{S} and/or α_2. We classify the contributions according to the source of CP violation:

(i) CP violation in mixing: Here, CP violation comes from $|q/p| \neq 1$ (as in $\mathrm{Re}(\epsilon)$ in $K \to \pi\ell\nu$). We identify two potentially significant contributions. The first is given by

$$\varepsilon_1^m = \frac{x^2}{4(1+x^2)}\left(\left|\frac{p}{q}\right|^2 - \left|\frac{q}{p}\right|^2\right)\Delta_{sf} = \mathcal{O}\left(\frac{x\Delta_{sf}\epsilon_{\mathrm{S}}}{1+x^2}\right). \tag{4.1}$$

This is the contribution discussed in [7, 8]. The size of this contribution depends crucially on

$$\Delta_{sf} \equiv \frac{\mathcal{N}_s(|A_{\widetilde{L}}|^2 + |\overline{A}_{\widetilde{L}^\dagger}|^2) - \mathcal{N}_f(|A_{\overline{L}}|^2 + |\overline{A}_L|^2)}{\mathcal{N}_s(|A_{\widetilde{L}}|^2 + |\overline{A}_{\widetilde{L}^\dagger}|^2) + \mathcal{N}_f(|A_{\overline{L}}|^2 + |\overline{A}_L|^2)}\,. \tag{4.2}$$

At zero temperature, $\Delta_{sf} = \mathcal{O}(\epsilon_{\rm S}^2)$, but for temperature at the time of decay that is comparable to the singlet sneutrino mass, $T_d \sim M$, we have $\Delta_{sf} \approx (\mathcal{N}_s - \mathcal{N}_f)/(\mathcal{N}_s + \mathcal{N}_f) = \mathcal{O}(1)$. The second contribution is given by (neglecting now corrections of order Δ_{sf})

$$\begin{aligned}
\varepsilon_2^m &= -\frac{x}{4(1+x^2)}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) \times \\
&\times \left[\left(\left|\frac{\overline{A}_{\widetilde{L}}}{A_{\widetilde{L}}}\right| + \left|\frac{A_{\widetilde{L}^\dagger}}{\overline{A}_{\widetilde{L}^\dagger}}\right|\right)\sin\delta_s\cos\phi_s - \left(\left|\frac{\overline{A}_{\overline{L}}}{A_{\overline{L}}}\right| + \left|\frac{A_L}{\overline{A}_L}\right|\right)\sin\delta_f\cos\phi_f\right] \\
&= \mathcal{O}\left(\frac{\epsilon_{\rm S}^2\alpha_2}{1+x^2}\right).
\end{aligned} \tag{4.3}$$

(ii) CP violation in interference of decays with and without mixing: Here, CP violation comes from $\arg(\lambda_X\lambda_{\overline{X}}) \neq 0$ (as in $S_{\psi K_S}$ in $B \to J/\psi K$ and similar to the mixing contribution to standard leptogenesis (see e.g. [14]), though mixing in the latter case is between different generations rather than between CP conjugate states). We identify the following potentially significant contribution:

$$\varepsilon^i = -\frac{y}{2}\left[\left(\left|\frac{\overline{A}_{\widetilde{L}}}{A_{\widetilde{L}}}\right| + \left|\frac{A_{\widetilde{L}^\dagger}}{\overline{A}_{\widetilde{L}^\dagger}}\right|\right)\sin\delta_s\sin\phi_s - \left(\left|\frac{\overline{A}_{\overline{L}}}{A_{\overline{L}}}\right| + \left|\frac{A_L}{\overline{A}_L}\right|\right)\sin\delta_f\sin\phi_f\right] = \mathcal{O}\left(\epsilon_{\rm S}^2\alpha_2\right). \tag{4.4}$$

(iii) CP violation in decay: Here, CP violation comes from $|A_X| \neq |\overline{A}_{\overline{X}}|$ (as in ${\rm Re}(\epsilon')$ in $K \to \pi\pi$ and as in the vertex contribution to standard leptogenesis). We identify the following potentially significant contribution:

$$\varepsilon^d = \frac{y}{2}\left(\left|\frac{\overline{A}_{\widetilde{L}}}{A_{\widetilde{L}}}\right| - \left|\frac{A_{\widetilde{L}^\dagger}}{\overline{A}_{\widetilde{L}^\dagger}}\right|\right)\cos\delta_s\cos\phi_s = \mathcal{O}\left(\epsilon_{\rm S}^2\alpha_2\right). \tag{4.5}$$

(iv) We also find a contribution that involves all three types of CP violation and is not necessarily sub-dominant:

$$\varepsilon^{mdi} = -\frac{x}{4(1+x^2)}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\left(\left|\frac{\overline{A}_{\widetilde{L}}}{A_{\widetilde{L}}}\right| - \left|\frac{A_{\widetilde{L}^\dagger}}{\overline{A}_{\widetilde{L}^\dagger}}\right|\right)\cos\delta_s\sin\phi_s = \mathcal{O}\left(\frac{\epsilon_{\rm S}^2\alpha_2}{1+x^2}\right). \tag{4.6}$$

We note that, apart from ε_1^m, all the contributions involve loop diagrams with gaugino exchange. The gaugino is playing a double role here. First, its mass provides a new physical CP violating phase. Second, the loop diagrams provide a strong phase. Consequently, direct CP violation becomes a possible source of the lepton asymmetry. Gaugino interactions do not violate lepton number, but they allow the lepton number violating time evolution of the heavy sneutrinos to contribute to ε_ℓ in new ways. Without gaugino interactions, indirect CP violation is the only significant source of soft leptogenesis [7, 8]. Direct CP violation can still be induced, but it involves higher powers of the Yukawa couplings and is therefore negligibly small.

5. The Size of ε_ℓ

In the previous section, we distinguished five potentially important contributions to ε_ℓ. These five contributions can be separated into three different classes:

$$\begin{aligned}
\varepsilon_\ell &= \varepsilon_1^m + \left(\varepsilon^i + \varepsilon^d\right) + \left(\varepsilon_2^m + \varepsilon^{mdi}\right), \\
\varepsilon_1^m &= \mathcal{O}\left(\frac{x\Delta_{sf}\epsilon_{\rm S}}{1+x^2}\right), \\
\varepsilon^i, \varepsilon^d &= \mathcal{O}\left(\epsilon_{\rm S}^2\alpha_2\right), \\
\varepsilon_2^m, \varepsilon^{mdi} &= \mathcal{O}\left(\frac{\epsilon_{\rm S}^2\alpha_2}{1+x^2}\right).
\end{aligned} \tag{5.1}$$

The generated baryon to entropy ratio is given by

$$\frac{n_B}{s} \simeq -\kappa 10^{-3}\varepsilon_\ell\,, \tag{5.2}$$

where $\kappa \lesssim 1$ is a dilution factor which takes into account the possible inefficiency in the production of the heavy sneutrinos or erasure of the generated asymmetry by lepton number violating scattering processes. Since observations determine $n_B/s \sim 10^{-10}$, any of the contributions in (5.1) would be significant only if it yields $|\varepsilon_\ell| \gtrsim 10^{-6}$. We now specify the conditions on the parameters whereby each of the three classes of contributions can be responsible for a successful leptogenesis. Since all the effects that we consider are related to supersymmetry breaking and therefore suppressed by powers of $\epsilon_{\rm S}$, soft leptogenesis can give significant effects only for $\epsilon_{\rm S} \gtrsim 10^{-6}$, that is,

$$M \lesssim 10^6\ m_{\rm SUSY} \sim 10^9\,{\rm GeV}\,. \tag{5.3}$$

In order that the singlet neutrino and sneutrino decay out of equilibrium, we should have a decay rate, $\Gamma = M|Y|^2/4\pi$, that is not much faster than the expansion rate of the Universe, $H = 1.66 g_*^{1/2} T^2/m_{\rm Pl}$ (g_* counts the effective number of spin degrees of freedom in thermal equilibrium; $g_* = 228.75$ in the SSM), at the time when the temperature is of order M:[1]

$$\frac{M}{|Y|^2} \gtrsim 3\times 10^{16}\,{\rm GeV}\,. \tag{5.4}$$

On the other hand, the sneutrino decay should occur before the electroweak phase transition, when sphalerons are still active, $\Gamma > H(T\sim 100\,{\rm GeV})$:

$$M|Y|^2 \gtrsim 3\times 10^{-13}\,{\rm GeV}\,. \tag{5.5}$$

[1] Taken at face value, eq. (5.4) implies an upper bound on the lightest neutrino mass of order $10^{-3}\,$eV, suggesting that the light neutrino masses are hierarchical. However, in the parts of our parameter space where $|\varepsilon_\ell| > 10^{-6}$, some wash-out effects can be tolerated, which would relax this bound.

Combining eqs. (5.3), (5.4) and (5.5), we learn that soft leptogenesis can give significant effects only for

$$10^{-11}\left(\frac{10^9\,\text{GeV}}{M}\right)^{1/2} \lesssim |Y| \lesssim 10^{-4}\left(\frac{M}{10^9\,\text{GeV}}\right)^{1/2}. \tag{5.6}$$

With such a small Yukawa coupling, the decay width is rather narrow,

$$\Gamma \lesssim 1\,\text{GeV}\left(\frac{M}{10^9\,\text{GeV}}\right)^2. \tag{5.7}$$

(i) The contribution from ε_1^m is of order $(x/(1+x^2))\Delta_{sf}\epsilon_{\rm S}$. For temperatures well below the mass M, the finite temperature contribution to Δ_{sf} is given by the following approximation ($n_{s,f} = (e^{M/(2T)} \mp 1)^{-1}$):

$$\Delta_{sf} \simeq \frac{(1+n_s)^2-(1-n_f)^2}{(1+n_s)^2+(1-n_f)^2} \approx 2e^{-M/(2T_d)}\,, \tag{5.8}$$

where T_d is the temperature at the time of decay. To obtain $|\varepsilon_\ell| \gtrsim 10^{-6}$ we must have

$$\frac{T_d}{M} \gtrsim \frac{1}{2\ln(2\epsilon_{\rm S}/10^{-6})}. \tag{5.9}$$

By using $\Gamma = H(T_d)$, this can be translated into an upper bound on $M/|Y|^2$:

$$\frac{M}{|Y|^2} \lesssim 4\times 10^{16}\,\text{GeV}[2\ln(2\epsilon_{\rm S}/10^{-6})]^2\,. \tag{5.10}$$

The lower (5.10) and upper (5.4) bounds define, for given M, a range for $|Y|$ and a range for Γ. Finally, we must have $x/(1+x^2) \gtrsim 10^{-6}/(\Delta_{sf}\epsilon_{\rm S})$. Taking into account that $x = 2|B|/(M\Gamma)$, for a given value of M we obtain an allowed range for B. Since the naive estimate is $|B| \sim Mm_{\rm SUSY}$, it is useful to write the allowed range for $|B|$ in units of $Mm_{\rm SUSY}$. We do so in figure 2. We conclude that ε_1^m can account for the observed baryon asymmetry under the following conditions:

(a) The mass of the lightest sneutrino is light enough, $M \lesssim 10^9$ GeV.

(b) The Yukawa couplings are small enough, $Y \lesssim 10^{-4}$. The lighter is M, the smaller the Yukawa coupling must be.

(c) The B parameter is well below its naive value, $|B|/(Mm_{\rm SUSY}) \lesssim 10^{-3}$. The lighter is M, the more suppressed the B coupling must be.

We note that the inclusion of three body decays [15] does not change the basic picture and, in particular, does not modify the estimate of Δ_{sf}. We prove this statement in appendix A.

(ii) The contribution from ε^d and ε^i is of order $\alpha_2\epsilon_{\rm S}^2$. Since $\alpha_2 \sim 10^{-2}$, we must have

$$M \lesssim 10^2 m_{\rm SUSY}\,. \tag{5.11}$$

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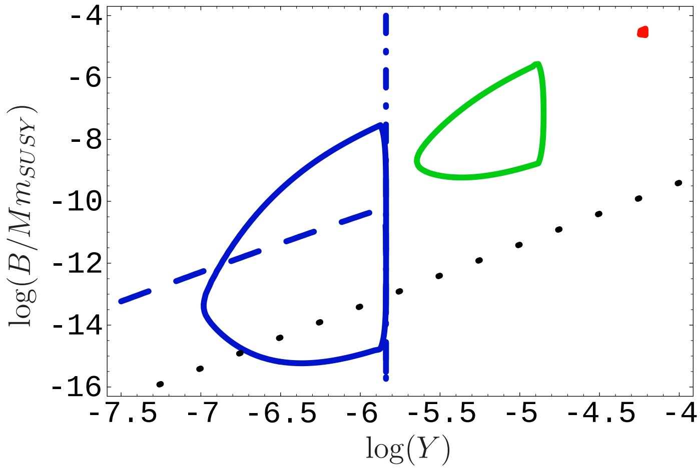

Figure 2: Regions in the $Y - B$ plane where $\Gamma < H(T = M)$ and $\varepsilon_\ell \gtrsim 10^{-6}$. We take $m_{\rm SUSY} = 10^3$ GeV. The approximation made in our calculations ($x > \epsilon_{\rm S}$) does not hold below the dotted line. (i) For $\varepsilon_\ell \sim \varepsilon_1^m$, the allowed regions are within the solid curves, for $M = 2 \times 10^8$ (right), 10^7 (middle) and 10^5 (left) GeV. (ii) For $\varepsilon_\ell \sim \varepsilon^d + \varepsilon^i$, the allowed region is to the left of the dash-dotted line, with $M = 10^5$ GeV. (iii) For $\varepsilon_\ell \sim \varepsilon_2^m + \varepsilon^{mdi}$, the allowed region is below and to the the left of the dashed curve, for $M = 10^5$ GeV.

Eq. (5.6) then requires

$$Y \lesssim 10^{-6}. \tag{5.12}$$

The region where this class of contributions can account for the observed baryon asymmetry is to the left of the dash-dotted line in figure 2. Note that $|B|$ is not constrained in this scenario. In particular, it can take its naive value, $|B| \sim Mm_{\rm SUSY}$, in which case $x = 2|B|/(M\Gamma) \gtrsim 10^{11}$, so that the sneutrino oscillation rate is much faster than its decay rate.

(iii) The contribution from ε_2^m and ε^{mdi} is of order $\alpha_2\epsilon_{\rm S}^2/(1+x^2)$. Consequently, the bound (5.11) on M and the bound (5.12) on Y apply. In addition, we must have $x \ggg 1$, which implies

$$\frac{B}{Mm_{\rm SUSY}} \lesssim \frac{M}{m_{\rm SUSY}} \frac{Y^2}{8\pi} \lesssim 10^{-11}\,. \tag{5.13}$$

This third class of contributions is never much larger than the second class. It may, however, be comparable if B is small enough. The region where this class of contributions is significant is to the left and below the dashed line.

We note that, since our calculations are performed with the assumption that $x \gg \epsilon_{\rm S}$, they should not be trusted for $B/(Mm_{\rm SUSY}) < Y^2/(8\pi)$, that is below the dotted line in figure 2.

6. Conclusions

Our main conclusions regarding the range of parameters where soft leptogenesis may be successful are the following:

1. Soft leptogenesis can be neglected for $M \gg 10^9$ GeV.

2. Soft leptogenesis can work for $M \gg 10^5$ GeV only if the Yukawa couplings have small values in a rather narrow range and if the B parameter is very small compared to its naive scale ($Mm_{\rm SUSY}$). (We comment on the possibility of *naturally* achieving $B \ll Mm_{\rm SUSY}$ in Appendix B.)

3. For $M \lesssim 10^5$ GeV there are several contributions from soft leptogenesis that could account for the observed baryon asymmetry. All the supersymmetry soft breaking terms can assume their natural values.

The main novel point of this paper is the realization that soft supersymmetry breaking terms give contributions to the lepton asymmetry that are related to CP violation in decays (ε^d of eq. (4.5)) and in the interference of decays with and without mixing (ε^i of eq. (4.4)). In contrast to CP violation in mixing (ε^m of eq. (4.1)), the oscillation rate needs not be comparable to the decay rate in order to have a significant effect. This is the reason that the B term can assume natural values. The new contributions to ε_ℓ are second order in supersymmetry breaking terms and further suppressed by a loop factor ($\sim \alpha_2 m_2 A/(M^2 Y)$) and are, therefore, significant only if M is not much higher than $10^2 m_{\rm SUSY}$.

The contribution to the lepton asymmetry related to CP violation in mixing (ε_1^m of eq. (4.1)), which was originally discussed in refs. [7, 8], requires thermal effects in order to be significant. In contrast, the new contributions discussed here (such as ε^i of eq. (4.4) and ε^d of eq. (4.5)) do not require thermal effects and, consequently, allow a non-thermal scenario of leptogenesis to work. Such a scenario would arise if, for example, sneutrinos were produced by inflaton decays (or if the sneutrino itself were the inflaton), and the temperature of the thermal bath at the epoch of decay is well below M (though above the electroweak scale so that sphalerons are still active).

Soft leptogenesis opens up the interesting possibility that the scale of the lightest singlet (s)neutrino mass (M) is not far above the electroweak scale. In contrast, standard leptogenesis cannot yield, in general, a large enough asymmetry for low M. The difference between the two scenarios lies in the different role of the Yukawa couplings. In both standard and soft leptogenesis, the condition for out of equilibrium decay associates a low scale M with tiny Yukawa couplings Y. In standard leptogenesis, the Yukawa couplings are the source of CP violation; therefore, small Y yield a small ε_ℓ. In soft leptogenesis, CP violation is induced by soft supersymmetry breaking terms and is not suppressed by small Y.

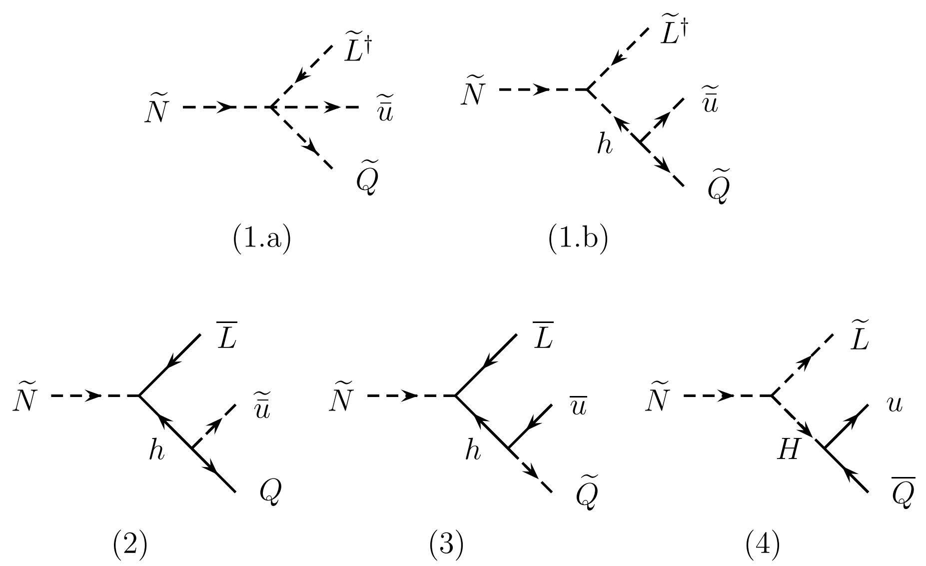

Figure 3: Three-body decay diagrams of a singlet sneutrino.

A. Three body decays

The H_u field has Yukawa couplings to neutrinos and to up quarks. The superpotential terms, $W = YNLH + Y_u Q\bar{u}H$, give a quartic scalar interaction term in the lagrangian,

$$\mathcal{L}_4 = YY_u^* \widetilde{N}\widetilde{L}\widetilde{Q}^\dagger \widetilde{\bar{u}}^\dagger + \text{h.c.}\,, \tag{A.1}$$

where $\widetilde{Q}$ is the scalar quark doublet and $\widetilde{\bar{u}}$ is the up-singlet. This coupling allows the three body decay mode, $\widetilde{N} \to \widetilde{L}^\dagger \widetilde{\bar{u}}\widetilde{Q}$. Since there is no similar quartic coupling of $\widetilde{N}$ to two fermions and one scalar, one may think that for the three body decays, the vanishing of $\Delta_{sf}^{(3)}$ (defined in (A.3)) in the supersymmetric limit is avoided, and a sizeable lepton asymmetry is induced even at zero temperature. This is, however, not the case, as we now explain.

We are considering contributions to the CP asymmetry of the form

$$\varepsilon^{(3)} = \frac{x^2}{4(1+x^2)} \left(\left|\frac{q}{p}\right|^2 - \left|\frac{p}{q}\right|^2 \right) \Delta_{sf}^{(3)}\,, \tag{A.2}$$

where

$$\Delta_{sf}^{(3)} \equiv \frac{\sum_{i=1}^{4} (-1)^{L_i+1} \mathcal{N}_i |A_i^{(3)}|^2}{\mathcal{N}_s |\overline{A}_L|^2 + \mathcal{N}_f |A_{\widetilde{L}}|^2}\,. \tag{A.3}$$

Here $A_i^{(3)}$ is the amplitude of a relevant final three body state with lepton number $L_i = \pm 1$. At zero temperature, all the three body phase space factors $\mathcal{N}_i$ are equal in the supersymmetric limit and, consequently, $\Delta_{sf}^{(3)} \propto \sum_{i=1}^{4} (-1)^{L_i+1} |A_i^{(3)}|^2$. The five tree level diagrams, leading to four different final states, are shown in figure 3. A straightforward calculation gives:

$$\begin{aligned}
\left|A_1^{(3)}\right|^2 &= \left|A_{1a}^{(3)} + A_{1b}^{(3)}\right|^2 = 2|Y|^2|Y_u|^2\frac{m_{23}^4}{(m_{23}^2-\mu^2)^2}\,,\\
\left|A_2^{(3)}\right|^2 &= 2|Y|^2|Y_u|^2\frac{m_{12}^2m_{23}^2}{(m_{23}^2-\mu^2)^2}\,,\\
\left|A_3^{(3)}\right|^2 &= 2|Y|^2|Y_u|^2\frac{m_{13}^2m_{23}^2}{(m_{23}^2-\mu^2)^2}\,,\\
\left|A_4^{(3)}\right|^2 &= 2|Y|^2|Y_u|^2\frac{M^2m_{23}^2}{(m_{23}^2-\mu^2)^2}\,,
\end{aligned} \tag{A.4}$$

where μ is the supersymmetric mass of the H_u supermultiplet, and $m_{ij}^2 = (k_i + k_j)^2$, with k_1, k_2, k_3 the momenta of, respectively, the final (s)lepton, the singlet up (s)quark and the doublet (s)quark. Then,

$$\begin{aligned}
\sum_{i=1}^{4}(-1)^{L_i+1}|A_i^{(3)}|^2 &= |A_1^{(3)}|^2 + |A_2^{(3)}|^2 + |A_3^{(3)}|^2 - |A_4^{(3)}|^2\\
&= 2|Y|^2|Y_u|^2\frac{m_{23}^2(M^2-m_{12}^2-m_{13}^2-m_{23}^2)}{(m_{23}^2-\mu^2)^2} = 0\,.
\end{aligned} \tag{A.5}$$

The last equation, that is the vanishing of the $\sum_{i=1}^{4}(-1)^{L_i+1}|A_i^{(3)}|^2$, holds in the supersymmetric limit, when the three final particles are massless. The result is that, in the supersymmetric limit, $\varepsilon_\ell^{(3)} = 0$. The vanishing of $\Delta_{sf}^{(3)}$ is lifted by finite temperature effects, similarly to the case of Δ_{sf}, but then the contribution of the three body states is small compared to the dominant two body ones.

If we assign lepton number $L = 0$ to the N-supermultiplet, then the quantities Δ_{sf} defined in eq. (4.2) and $\Delta_{sf}^{(3)}$ defined in eq. (A.3) are the asymmetries between $\Delta L = +1$ and $\Delta L = -1$ decay rates. Then, the vanishing in the supersymmetric limit of Δ_{sf} and $\Delta_{sf}^{(3)}$, demonstrated explicitly in our work, becomes understandable on general grounds and generalizes to n-body states for any n. In a single generation framework and in the absence of supersymmetry breaking, singlet neutrino decay rates to leptons and antileptons must be equal. Then, by supersymmetry, this should hold also for singlet sneutrinos.

B. On the naturalness of $B = 0$

We consider the following superpotential terms:

$$W = MNN + YNLH\,, \tag{B.1}$$

and SUSY breaking terms,

$$\mathcal{L} = B\widetilde{N}\widetilde{N} + A\widetilde{N}\widetilde{L}H\,. \tag{B.2}$$

In the absence of these terms, there are four additional flavor conserving global U(1) symmetries: $\mathrm{U}(1)_N \times \mathrm{U}(1)_L \times \mathrm{U}(1)_H \times \mathrm{U}(1)_R$, with the following charge assignments:

$$\begin{aligned} &N(1,0,0,0), &&\widetilde{N}(1,0,0,1)\,,\\ &L(0,1,0,0), &&\widetilde{L}(0,1,0,1)\,,\\ &h(0,0,1,0), &&H(0,0,1,1)\,. \end{aligned} \tag{B.3}$$

Selection rules for the symmetries may be used if M, Y, A and B are treated as spurions with charges assigned to compensate those of the fields:

$$M(-2,0,0,0)\,,\qquad Y(-1,-1,-1,-1)\,,\qquad A(-1,-1,-1,-3)\,,\qquad B(-2,0,0,-2)\,. \tag{B.4}$$

To understand the consequences, it is simpler to examine the charges of the spurions under $\mathrm{U}(1)_{N-L} \times \mathrm{U}(1)_{2R-3(L+H)} \times \mathrm{U}(1)_{2R-(L+H)} \times \mathrm{U}(1)_{L+H}$:

$$M(-2,0,0,0)\,,\qquad Y(0,+4,0,0)\,,\qquad A(0,0,-4,0)\,,\qquad B(-2,-4,-4,0)\,. \tag{B.5}$$

We learn that setting $B=0$ does not add a symmetry to the lagrangian. Consequently, B is *additively* renormalized. However, setting B and any other of the three couplings, M, Y or A, to zero is natural.

We can therefore think of a three generation framework where, for example, $Y=0$ because of a supersymmetric Froggatt-Nielsen symmetry [16, 17]. Then $B=0$ is natural. When the FN symmetry is spontaneously broken, B can be naturally suppressed:

$$B \propto AMY^\dagger \ll AM\,. \tag{B.6}$$

Of course, a Froggatt-Nielsen symmetry can also induce $A \sim m_{\rm SUSY} Y \ll m_{\rm SUSY}$, leading to further suppression of B compared to $M m_{\rm SUSY}$. Both the additive renormalization, and the suppression factors in (B.6) are manifest in the RGE [18]:

$$16\pi^2 \frac{d}{dt} B = MY^* A\,, \tag{B.7}$$

If, however, B is radiatively generated, as in (B.7), the phase ϕ_N vanishes at this order. At two loops, there will be a contribution to B that depends on m_2, but then $\phi_N \sim \alpha_2$.

Acknowledgments

We thank Lance Dixon, Gian Giudice, Riccardo Rattazzi and Yael Shadmi for useful discussions. This work was supported in part by a grant from Fundación Antorchas/Weizmann. The research of Y.G. and Y.N. is supported by a Grant from the G.I.F., the German-Israeli Foundation for Scientific Research and Development, and by the Einstein Minerva Center for Theoretical Physics. The work of Y.G. is supported by the United States-Israel Binational Science Foundation through grant No. 2000133, by the Israel Science Foundation under grant No. 237/01, by the Department of Energy, contract DE-AC03-76SF00515 and by the Department of Energy under grant No. DE-FG03-92ER40689. Y.N. is supported by the Israel Science Foundation founded by the Israel Academy of Sciences and Humanities, by EEC RTN contract HPRN-CT-00292-2002, by the Minerva Foundation (München), and by a grant from the United States-Israel Binational Science Foundation (BSF), Jerusalem, Israel. E.R. is partially supported by a John Simon Guggenheim Foundation fellowship.

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